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M.Sc (Mathematics)

Semester - IV

Paper Code - CC Math 401

Subject - Numerical solution of (ODE/ PDE)

Short Questions.

(1) Classify the following equations:

(i) $$\frac{\partial^2 f}{\partial x^2} + 2\frac{\partial^2 f}{\partial x \partial y} + 4\frac{\partial^2 f}{\partial y^2} = 0$$

(ii) $$(1+x^2)\frac{\partial^2 u}{\partial x^2} + (5+2x^2)\frac{\partial^2 u}{\partial x \partial t} + (4+x^2)\frac{\partial^2 u}{\partial t^2} = 0.$$

(2) Explain Bender Schmidt method for the Soln of one dimensional heat equation $\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$.

(3) Explain the Crank-NiColson method for the Soln of One-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

(4) Explain ADE (Alternating Direction Explicit) method for the Solution of $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$.

(5) Explain the implicit scheme for the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{—(1)}$$

Subject to the initial Conditions $u = f(x)$, $\frac{\partial u}{\partial t} = g(x)$, $0 \le x \le 1$ at $t=0$

and the boundary conditions $u(0,t) = \phi(t)$, $u(1,t) = \psi(t)$

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Long Questions

① Solve the boundary value problem $u_t = u_{xx}$ under the conditions $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin \pi x$, $0 \le x \le 1$ using Schmidt method. Take $h = 0.2$ and $\alpha = \frac{1}{2}$.

② Find the values of $u(x,t)$ satisfying the parabolic equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$

and the boundary conditions $u(0,t) = 0 = u(8,t)$ and $u(x,0) = 4x - \frac{1}{2}x^2$ at the points $x = i;\ i = 0,1,2 \cdots 7$ and $t = \frac{1}{8}j;\ j = 0,1,2 \cdots 5$.

③ Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

subject to the initial conditions $u(x,y,0) = \sin 2\pi x \sin 2\pi y$, $0 \le x, y \le 1$ and the conditions $u(x,y,t) = 0$, $t > 0$ on the boundaries, using ADE method with $h = \frac{1}{3}$ and $\alpha = \frac{1}{8}$. Calculate the results for one time level.

④ Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking $\Delta x = 1$

upto $t = 1.25$. The boundary conditions are $u(0,t) = u(5,t) = 0$,

$u_t(x,0) = 0$ and

$u(x,0) = x^2(5-x)$.

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(5) Solve $y_{tt} = y_{xx}$ upto $t = 0.5$

with a spacing of 0.1 subject to

$y(0,t) = 0$, $y(1,t) = 0$,

$y_t(x,0) = 0$ and $y(x,0) = 10 + x(1-x)$.

(6) Find the solution of the initial boundary value problem:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1 \text{ ; subject to the}$$

initial Conditions $u(x,0) = \sin \pi x$, $0 \le x \le 1$,

$\left(\frac{\partial u}{\partial t}\right)(x,0) = 0$, $0 \le x \le 1$ and the

boundary Conditions $u(0,t) = 0$,

$u(1,t) = 0$, $t > 0$; by using in

the (a) the explicit scheme

(b) the implicit scheme.